

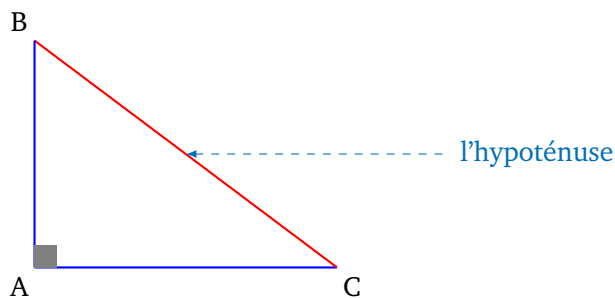
Chapitre 2

Le théorème de Pythagore

1. Vocabulaire

Définition 1 : hypoténuse

Dans un triangle **rectangle**, le côté opposé à l'angle droit est appelé **hypoténuse**.



Remarque

Dans un **triangle rectangle**, l'hypoténuse est le **plus grand** des trois côtés.

2. Le théorème de Pythagore

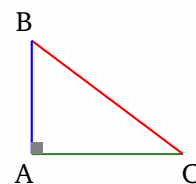
Propriété 1 : L'égalité de Pythagore

Si un triangle est **rectangle**, alors le carré de la longueur de l'hypoténuse est égal à la somme des carrés des longueurs des deux autres côtés.

Exemple

Le triangle ABC ci-contre est rectangle en A.
L'égalité de Pythagore est donc satisfaite et on a

$$BC^2 = AB^2 + AC^2$$



Remarque

L'égalité de Pythagore permet de calculer la longueur d'un côté d'un **triangle rectangle** lorsque l'on connaît les mesures des deux autres côtés.

3. Applications

Méthode 1 : calculer la longueur de l'hypoténuse

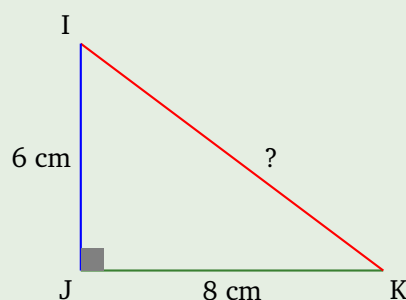
Le triangle IJK est rectangle en J, donc d'après le théorème de Pythagore

$$KI^2 = IJ^2 + JK^2.$$

$$\text{Donc } KI^2 = 6^2 + 8^2 = 36 + 64 = 100.$$

La longueur KI est un nombre positif qui, élevé au carré, vaut 100.

$$\text{Or } 10^2 = 100, \text{ donc } \boxed{KI = 10 \text{ cm.}}$$



Méthode 2 : calculer la longueur d'un côté de l'angle droit

Soit EFG un triangle rectangle en G tel que EF = 8 cm et EG = 3 cm. Calculer FG.

Solution

Le triangle EFG est rectangle en G, donc d'après le théorème de Pythagore

$$EF^2 = EG^2 + FG^2.$$

Ainsi

$$8^2 = 3^2 + FG^2 \quad \text{ou encore} \quad 64 = 9 + FG^2.$$

Par conséquent

$$FG^2 = 64 - 9 = 55.$$

La valeur exacte de FG est appelée **racine carrée** de 55 et on la note $\sqrt{55}$.

Elle est comprise entre 7 et 8 car $49 = 7^2 \leq 55 \leq 8^2 = 64$.

Avec les touches **2nde** et **x²** de la calculatrice, on obtient

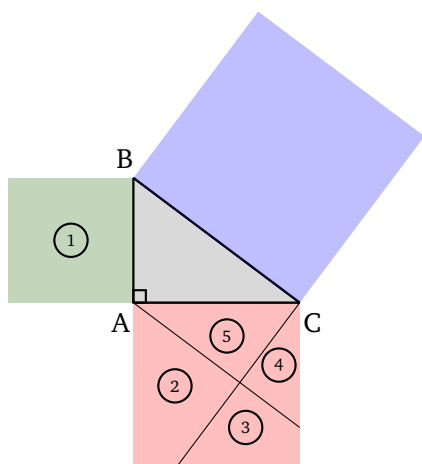
$$\sqrt{55} \approx 7,416198487.$$

Donc

$$\boxed{FG \approx 7,4 \text{ cm.}}$$

Il s'agit d'une valeur approchée au dixième près de la longueur FG.

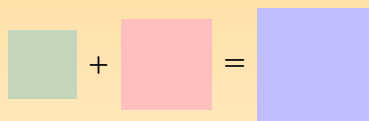
4. Une preuve du théorème de Pythagore



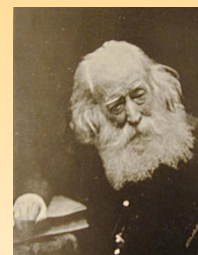
- Découper avec précision les pièces 1, 2, 3, 4 et 5 ;
- à l'aide de ces cinq pièces, reconstituer le grand carré ;
- traduire cette reconstitution par une égalité avec les aires des carrés.

Puzzle de Perigal

Henry Perigal (1801 - 1898) : mathématicien et astronome amateur britannique, connu pour sa démonstration du théorème de Pythagore par dissection.



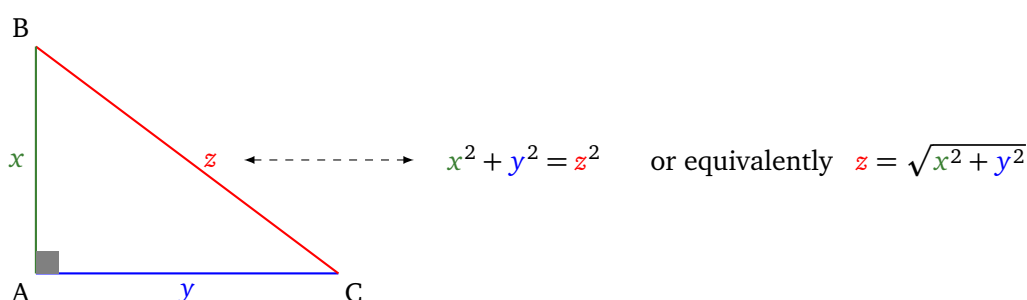
$$a^2 + b^2 = c^2$$



5. Le grand théorème de Fermat (en anglais)

1. The Pythagorean theorem

Pythagorean theorem, the well-known geometric theorem gives a relationship between the three sides of a right triangle. It states that the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides. In familiar algebraic notation, $x^2 + y^2 = z^2$ (see the drawing below). Although the theorem has long been associated with Greek mathematician-philosopher Pythagoras (c. 570–500/490 BCE), it is actually far older. Four Babylonian tablets from circa 1900–1600 BCE indicate some knowledge of the theorem, with a very accurate calculation of the square root of 2 (the length of the hypotenuse of a right triangle with the length of both legs equal to 1) and lists of special integers known as **Pythagorean triples** that satisfy it (e.g., 3, 4, and 5 ; $3^2 + 4^2 = 5^2$). The theorem is mentioned in the Baudhayana Sulba-sutra of India, which was written between 800 and 400 BCE. Nevertheless, the theorem came to be credited to Pythagoras. It is also proposition number 47 from Book I of Euclid's Elements.



2. Pierre de Fermat's Last Theorem

The 17th century mathematician Pierre de Fermat created the Last Theorem while studying Arithmetica, an ancient Greek text written in about AD 250 by Diophantus of Alexandria. This was a manual on number theory, the purest form of mathematics, concerned with the study of whole numbers, the relationships between them, and the patterns they form.

The page of Arithmetica which inspired Fermat to create the Last Theorem discussed various aspects of Pythagoras' Theorem. In particular, Arithmetica asked its readers to find solutions to Pythagoras' equation, such that x, y, and z could be any whole numbers, **except zero**. For example, $3^2 + 4^2 = 5^2$ (i.e. $9 + 16 = 25$) or $5^2 + 12^2 = 13^2$ (i.e. $25 + 144 = 169$). Fermat must have been bored with such a tried and tested equation, and as a result he considered a slightly mutated version of the equation :

$$x^3 + y^3 = z^3$$

The equation is now said to be to the power 3, rather than the power 2. Surprisingly, the Frenchman came to the conclusion that among the infinity of numbers there were none that fitted this new equation. Whereas Pythagoras' equation had many possible solutions, Fermat claimed that his equation was insoluble.

Fermat went even further, believing that if the power of the equation were increased further, then these equations would also have no solutions :

$$\begin{aligned}
 x^3 + y^3 &= z^3 \\
 x^4 + y^4 &= z^4 \\
 x^5 + y^5 &= z^5 \\
 x^6 + y^6 &= z^6 \\
 &\dots\dots\dots \\
 x^n + y^n &= z^n
 \end{aligned}$$

To back up his theorem he had developed an argument or mathematical proof, and following the first marginal note he scribbled the most tantalising comment in the history of mathematics :

The missing proof of Fermat

I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain.

3. Peer review

In mathematics, proof is the process of carefully reasoned steps that create new knowledge based on the axioms, or based on other proofs which previously have been based on the axioms. However, no mathematical result enters the realm of mathematics until it becomes public knowledge : it must undergo **peer review**, just as do the knowledge claim of the sciences.

A good example of peer review at work is the rejection, for almost four centuries, of all attempted proofs for what came to be known as Fermat's Last Theorem.

Yet the theorem attracted huge numbers of amateur mathematicians and puzzle-solvers, eager to win a prize that had been offered and to make history as the one who solved the famous FLT ! A professor in Germany, responsible for evaluating proofs submitted for the prize, found he could get little of his own work done with all the amateur proofs arriving in the post. His solution was to hand the proofs to his graduate students to check, along with printed cards of rejection.

Dear,

Thank you for your manuscript on the proof of Fermat's Last Theorem.

The first mistake is on:

Page..... Line.....

This invalidates the proof.

Professor E. M. Landau

.....

4. Andrew Wiles : the man who (re)discovered the missing proof

A good example of peer review at work is the rejection, for almost four centuries, of all attempted proofs for what came to be known as Fermat's Last Theorem.

In 1963 a 10-year old boy borrowed a book from his local library in Cambridge, England. The boy was Andrew Wiles, a schoolchild with a passion for mathematics, and the book that had caught his eye was 'The Last Problem' by the mathematician Eric Temple Bell. The book recounted the history of Fermat's Last Theorem, the most famous problem in mathematics, which had baffled the greatest minds on the planet for over three centuries.

There can be no problem in the field of physics, chemistry or biology that has so vehemently resisted attack for so many years. Indeed E.T. Bell predicted that civilisation would come to an end as a result of nuclear war before Fermat's Last Theorem would ever be resolved. Nonetheless young Wiles was undaunted. He promised himself that he would devote the rest of his life to addressing the ancient challenge.

The English mathematician presented a proof of the Shimura-Taniyama-Weil conjecture in 1993 (a 150 page-paper). An error was found in this proof, however, but, with help from his former student Richard Taylor, Wiles finally devised a proof of Fermat's Last Theorem, which was published in 1995 in the journal Annals of Mathematics.